Written re-exam on Lab-on-a-Chip course, Spring Semester 2009
Aug 26th, 2009

Examination time: 4 hours (9am – 1pm)
Allowed means: MYO "Fundamentals of Fluid Mechanics", lecture slides and a calculator (no computer or PDAs, please)
Complete solution should include all equations calculated down to a numerical answer. Numerical answer alone is not counted as a solution.
Some useful constants for your problems are listed in the end of the exam paper

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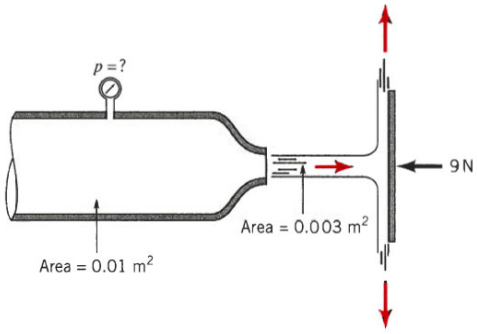
Problem 1. A circular stream of water flowing down from a faucet is observed to taper from a diameter 20mm to 10mm in a distance of 50mm. Determine the flow rate.

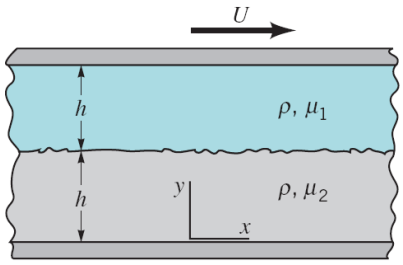
Problem 2. Air flows into the atmosphere from a nozzle and strikes a vertical plate. A horizontal force of 9N is required to hold the plate in place. Determine the reading on the pressure gauge. Assume the flow to be incompressible and frictionless..



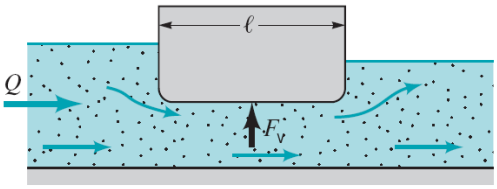
Problem 3. The two-dimensional velocity field for an incompressible Newtonian fluid is described as:

$\vec{V} = (12xy^2 - 6x^3)\vec{i} + \left(18x^2y - 4y^3\right)\vec{j}$

a) write the relationship for acceleration and determine it at the point with coordinates (0.5m, 1.0m)
b) Determine the stresses $\sigma_{xx}, \sigma_{yy}, \tau_{xy}$ at the same point if the pressure at this point is 6kPa and the fluid is glycerin at 20ºC.



Problem 4. Two immiscible incompressible viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal plates. The bottom plate is fixed and the upper plates moves with a constant velocity U. Determine the velocity at the interface using Navier-Stokes equation. Assume laminar flow and no pressure gradient in x direction.



Problem 5. Water flowing under the obstacle puts a vertical force $\mathbf{F_v}$ on the obstacle. This force is assumed to be a function of flow rate **Q**, the density of water **ρ**, the acceleration of gravity **g**, and a length ***l***, that characterizes the size of obstacle.
a) Find equation for the force using dimensional analysis and the above mentioned parameters.
b) A 1/20 scale model is built. If the prototype flow rate is 28 m^3/s, what flow rate is required in the model to be similar.

Problem 6. An experimentalist wants to achieve a water flow of 20 μl/min. He has a pressure regulator capable of applying 1 atm of gauge pressure and tubing of 0.125, 0.25 and 0.5mm diameter. What length of each type of tubing would be required to achieve the required flow rate?

List of constants:
Density of water 1000 kg/m^3;
Viscosity of water $1 \cdot 10^{-3}$ Pa*s
Density of glycerin 1260 kg/m^3,
Viscosity of glycerin 1.5 Pa*s